RC LARGE DISPLACEMENTS: OPTIMIZATION APPLIED TO EXPERIMENTAL RESULTS

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ABSTRACT

A large number of telecommunication towers were installed during the implantation of cellular telephony services in Brazil. Some of those towers presented problems as excessive displacements, residual displacements, cracking and some accidents happened. On the other hand, the computation of large displacements in slender reinforced concrete structures is a very difficult task as the flexural stiffness of the sections changes continuously as the bending moment increases, due to the very non-linear material behavior of concrete, involving such phenomena as formation of cracks and plastification. The goal of this paper is to present some initial results of the application of optimization techniques to experimental data relative to the determination of the effective bending stiffness of transverse sections of reinforced concrete structures. The objective is to determine parameters of reduction of the stiffness of unstressed sections for the correct calculation of the displacements of those structures. The results of a test with a reinforced concrete tower for telecommunications of 30 m of length, circular ring cross section with 50 cm diameter, were used. For several cross sections along the axis of this structure the effective stiffness was computed. For analysis purposes, the structure was discretized and the differential equation of the elastic line integrated to obtain the rotations and displacements. The values of the effective stiffness of the cross sections were obtained using optimization techniques. The effective stiffness is presented in graphs as function of the solicitation level (the ratio between the characteristic bending moment and the ultimate moment of the cross section). The section where the largest stiffness loss happened is the section that indeed collapsed in a real similar structure. Directions for future researches are presented.

Keywords: effective bending stiffness, experimental results, large displacements, optimization, reinforced concrete.

1 Introduction and Motivation

During the implantation of the cellular telephony services in Brazil, more than 10,000 towers for telecommunication were designed, fabricated and erected, from 1991 to 2001. From these, at least 2,000 are RC (reinforced concrete) towers. Only one company, Scac Fundações e Estruturas Ltda., which produces pre-fabricated centrifuged concrete poles, produced and installed around 1,500 towers in this period of time. In Fig. 1 we see a graph showing the number of towers that Scac erected by year. Brazil, that had no cellular phone services at the beginning of 90’s, reached the mark of 25.8 million installed lines in 2001. The implantation planning had a very ambitious schedule, in the sense that a large number of towers had to be erected in short period of time. At the beginning of the 90's, there were few companies and technicians able to attend the demand. Companies specialized in others products adapted their production line to the telecommunication market, which was quite promising.

In this context, structural designers, most of them of specialized in other applications, adapted their current mathematical models to design towers for telecommunication. The mathematical models adopted by most of them were based on static models, computing the wind load, the main loading acting on that type of structures, as static loads. In other words, the dynamic effects of the wind, besides considerations on terrain topography and roughness, were not computed for most of the installed structures.

In a short time, the effect of this very disturbed process came to surface: excessive displacements, residual displacements, cracking and, unfortunately, some accidents. Those were the reflexes of an extremely chaotic process. The authors participated of several studies related to the behavior, static or dynamic, of those steel and concrete structures. In this paper, the authors describe some results obtained in numerical analysis of results of tests accomplished with one 32 m long RC telecommunication tower.

On the other hand, one of the pertinent verifications in structural design is to check if the computed displacements are inside certain limits given by codes. In RC structures, in function of the specificities of this product, such as cracking and cross section stiffness reduction, this calculation becomes sometimes imprecise and, in those cases, the installed structures may present displacements larger than foreseen. In structural design, the cracking phenomena in RC structures are interpreted as a physical non-linearity. In other words, the stiffness depends on the solicitations.

The methodology of this work bases on comparing the displacements measured in tests in RC structures with data obtained integrating the elastic line. The quadratic error between the real displacements measured in tests and those provided by the elastic line integration is minimized using optimization techniques. The objective function is the quadratic error, while the design variables are the effective stiffness in each section. Constraints are imposed on the maximum and minimum values of the effective stiffness.

As a contribution to the studies found in literature on the determination of effective stiffness in RC cross sections, here we intend: i) to develop a procedure for the determination of the effective stiffness and to determine an equation involving the effective stiffness and the bending moment value in a certain section; ii) to explore the practical and theoretical aspects of the stiffness loss determination, checking if a relation exists between the sections where the larger losses happen and the sections that indeed collapse in a real similar structure.

2 A Brief Review of Literature

The two main tools to be used in this work are concepts of RC structures design and optimization techniques. For the RC analysis we adopt usual Brazilian methods, as presented by Brazilian National Codes NBR-6118-03 (ABNT [3]). A review of the specialized literature indicates that the effective stiffness of a RC cross-section of a beam depends on the bending moment value, as well as the arrangement of the reinforcement and the properties of material components. An equation proposed by Dan E. Branson in 1963 (Branson [6]) for the calculation of the effective stiffness was incorporated in ACI-318-71 (ACI [4]) and recently in NBR-6118-03 (ABNT [3]). Several researchers and designers have used the Branson equation to compute the displacements of RC beams. In this work, inspired in Branson's, we present another formulation to obtain the equations to represent the effective bending stiffness as function of the bending moment, using optimization techniques.

With respect to the optimization process, we use the Augmented Lagrangian Method (Chahande and Arora [9]). To render the constrained optimization problem into an unconstrained optimization problem, a Lagrangian functional is created. The objective and constraint functions are associated to Lagrange multipliers and penalty parameters. A sequence of Lagrangians is developed by properly varying the multipliers and penalty parameters. The minimum value of the Lagrangian function converges to the minimum of the optimization problem with constraints. In the cases analyzed in this work, the coefficients of the effective stiffness equations of the chosen cross-sections are the design variables for the error minimization problem. The error between the numerical and displacements measured in tests is the objective function, while the maximum and minimum values of the effective stiffness are the constraints. The line search algorithms we need to use in the process demand both the objective and constraints functions to be differentiable with respect to the design variables. In a certain stage of the calculation process, least square method is used (Arora [5]).

Silva and Brasil [10, 11] and Brasil and Silva [7, 8] used optimization techniques to compute the effective bending stiffness of slender RC structures as function of the bending moment value was. They presented results of experiments performed in several sample structures. They defined optimization problems where the design variables were the coefficients of some trial effective stiffness equations. The objective function was the quadratic error between the displacements measured in the experiments and those given by the integration of neutral line equation. They solved the optimization problems and obtained the effective stiffness equations for several kinds of structures. They considered several different functions to represent the effective stiffness parameter such as polynomial and trigonometric equations. In a paper by Silva and Brasil [11] a nonlinear dynamic analysis based on experimental results of a RC telecommunication tower was carried out using the discrete dynamic model given by NBR-6123-87 (ABNT [1]). First, they described some tests accomplished and results obtained for similar structures (Silva and Brasil [10]). They considered two different equations to represent the effective stiffness parameter: a linear one and another cubic equation. Once the effective stiffness equations and other information are known, they accomplished a nonlinear static analysis, under the mean wind velocity, considering the effective bending stiffness as a function of the bending moment value in each iteration of the P-Delta method. Considering the effective stiffness obtained in the final iteration of the P-Delta method they applied the discrete model given in NBR-6123-87 (ABNT [1]) to accomplish the dynamic analysis considering the floating wind velocity. They considered that the structure, under the floating wind velocity, vibrates around the equilibrium position given by the mean wind velocity and P-Delta method. Finally, they computed the sum of nonlinear static and linear dynamic bending moments. They compared the values obtained from dynamic analysis with those obtained from linear static analysis and those obtained in experimental work. The displacement values obtained in the proposed dynamic analysis were up to 40 % lager than those given by a static linear analysis. The tests results showed that the cross-section ultimate moment is up to 40 % larger than that given by NBR-6118-03 (ABNT [3]).

3 Tests Accomplished

A series of tests were conducted at the plant of SCAC Fundações e Estruturas Ltda. at São Paulo, Brazil. They were carried out on two centrifuged RC 30 and 40 *m* long structures. In this work we are going to present only the results of the 30 *m* long structure. The 30 *m* long structure presents circular cross-section with 50 *cm* diameter, thickness vary from 8 to 15 *cm* and 2 *m* of embedment. The geometrical and steel area data are shown in Table 1. In that table we have, in sequence, the sections numbering, the height above ground, the external diameter (ϕ_{ext}), the thickness (thick.), the total area of cross-section (A_{total}), the total longitudinal steel area (A_s), the total moment of inertia of cross-section (I_{total}), including the steel and concrete inertia, and the minimum value to be adopted by the parameter of effective stiffness w (w_s). The 30 *m* structure was designed and fabricated in three modules with lengths of 10 *m* for top module and 11 *m* for others (Fig. 2). Flanges installed at the extremities of them bolt these modules.

Once the structural designs were known, an appropriate planning of the several tests was prepared. The tests had the objective to evaluate the structures at the utilization limit state and ultimate limit state. The following parameters were measured during the tests: applied loads; displacements; occurrence and opening of cracks; residual displacements. The tests were performed with the structures in horizontal position. Fig.s 2 to 4 illustrates the test rig used for the 30 *m* long structure. Static loads were applied to the pole tip to produce bending moment and shear load along its axis. The loads were applied in the perpendicular direction to the structure axis. A stretched steel wire installed right above the structure materialized a fixed reference axis. Starting from the steel wire the displacements were measured at every 5 *m* of increment along the pole length. Movable supports were installed along the length of the structure to minimize forces due to self-weight and friction with ground (Fig. 4). The structure supports were designed to a load three times larger than its Brazilian Code predicted ultimate load. The apparatus was also long enough to provide appropriate embedment. Loads were applied gradually and after their stabilization the measured parameters were read. The intensity of the applied loads started at 5% of the ultimate Code based top load and increased gradually along the test stages to reach the collapse load of the structures. The characteristics of the structures, as well as the parameters values measured in the tests, mainly those related to the displacements and applied loads, are used in numerical simulations later. The loads applied and displacements measured in tests are shown in Table 2.

4 Introduction to the Determination of the Effective Stiffness

This formulation provides one equation for each section. So for one 30 *m* long structure, discretized in each one meter, it provides 31 equations. This formulation is very important because it can give information specifically about the behavior of one given section. For example, Silva and Brasil [10] and Brasil and Silva [8] showed that during a failure test in a 30 m long structure, the section that present the largest loss of stiffness is the section that really collapse when the structure failure occurs.

Consider the differential neutral line equation given by

$$EI_{EF}(z)v(z)'' = -M_k(z)\,, \tag{1}$$

where $I_{EF}(z)$, $v(z)$ and $M_k(z)$ are respectively the effective moment of inertia, the displacement and the characteristic bending moment of a transverse section with abscissa z. The concrete elasticity modulus (E = 41,4 *GPa*) is calculated as a function of f_{ck} = 45 *MPa* (characteristic compressive resistance of 28 days old concrete), as given by NBR-6118-78 (ABNT [2]) Brazilian Code. In the cases analyzed in this work, boundary conditions are $v(0) = 0$ and $v'(0) = 0$. The one-dimensional domain is the set $D = \{z \in \Re \mid 0 \le z \le L\}$. Dividing D in n segments, the discretized structure domain will now be $D_n = \{z \in \Re \mid z = z_0, \quad z_1, \quad \cdots \quad z_i, \quad \cdots \quad z_n\}$.

For each point z_i, $i > 0$, in this domain we compute $v_i = v(z_i)$, $v'_i = v'(z_i)$ and $v''_i = v''(z_i)$ using the following integration scheme:

$$\begin{aligned} v''_i &= -\frac{M_k(z_i)}{EI_{EF}(z_i)} \\ v'_i &= v'_{i-1} + \frac{v''_i + v''_{i-1}}{2} h \\ v_i &= v_{i-1} + \frac{v'_i + v'_{i-1}}{2} h \end{aligned} \tag{2}$$

where $h = z_i - z_{i-1}$. For each section z_i we consider the value of the effective moment of inertia

$$I_{EF}(z_i) = w_i\, I(z_i), \tag{3}$$

where $I(z_i)$ is the total homogenized moment of inertia (including concrete and homogenized steel) of the cross section z_i, and w_i is the portion of $I(z_i)$ that will actually resist the bending moment at each section. The total bending stiffness is defined as $EI(z_i)$, while the effective bending stiffness is $EI_{EF}(z_i)$. As I_{EF} depends on z_i and w_i,, from now on we will denote $I_{EF}(z_i,w_i)$. As a consequence, $v_i = v(z_i,\boldsymbol{w})$, where $\boldsymbol{w} = [w_0 \quad w_1 \quad \cdots \quad w_i \quad \cdots \quad w_n]^T$.

During the test the real displacements $v_r(x_i)$ were measured in some m sections of the structure under a certain loading. The absolute quadratic approximation error between the displacements given by Eq. (2) and those measured during the tests is the sum of m terms:

$$E_q(\boldsymbol{w}) = \frac{1}{2}\sum_i \left[v(z_i,\boldsymbol{w}) - v_r(z_i)\right]^2 . \tag{4}$$

We can formulate the following optimization problem. Determine $\boldsymbol{w} \in \Re^{n+1}$ that minimize the objective function

$$f(\boldsymbol{w}) = E_q(\boldsymbol{w}), \tag{5}$$

subjected to constrains:

$$w_s(z_i) \leq w_i \leq 1, \quad \text{para} \quad i = 0,1,...,n \tag{6}$$

where $w_s(z_i) = E_sI_s(z_i)/EI(z_i)$ (Table 1), where I_s and E_s = 210 GPa are, respectively, the steel moment of inertia and the elasticity modulus of the longitudinal reinforcement in section i. Solving the problem given by Eq. (5) and (6), we obtain the stiffness ratios $\boldsymbol{w}$ for a certain loading. The same calculations for all loading hypotheses will render a relationship between w_i and $M_k(z_i)/M_u(z_i)$, where M_u is the ultimate Code based moment computed according to NBR-6118-78 (ABNT [2]) Brazilian code, and

$$x = M_k(z_i)/M_u(z_i), \tag{7}$$

is the bending moment level in section z_i. As we noted earlier, we are going to show the analysis accomplished for 30 m long structure. The structures were discretized in each one meter. Thus, the domain is $D_{30} = \{z \in \Re \mid z = 0, \quad 1, \quad \cdots, \quad 30\}$, in meters. The points results for section $i = 12$ are shown in Fig. 5.

From graphs, we observe that the points are distributed around a tendency curve. Applying the least square method (Arora [5]) for the several points $[M_{ki}/M_{ui}, w_i] \equiv (x_i, y_i)$ and approximating by the following functions:

$$\begin{aligned} &y = ax + b \quad - && \text{Linear} \\ &y = ax^2 + bx + c && \text{Quadratic} \\ &y = ax^3 + bx^2 + cx + d && \text{Cubic} \end{aligned} \tag{8}$$

we obtain the values of coefficients *a*, *b*, *c* and *d*. In Fig. 6 we show the tendencies obtained for the points of sections showed in Fig. 5.

The mean values of coefficients *a*, *b*, *c* and *d* for sections that resisted at least $0.8M_u$ as well as the mean value of the absolute quadratic error corresponding to each approximation function in least square method were determinate. The obtained results are shown in Table 3. We considered the limit $0.8M_u$ because usually the structures are designed to work until this bending moment value and so using these data we can obtain the curves up to values that really happen in practice. There are cases when special overloads not foreseen in design happens, as, for example, the shock of a vehicle, leading to bending moments values larger than M_u. It is very important to the designer to know the behavior of this kind of structures near to the failure, because he can explore this aspects to reinforce an existing structure, or to design a new one.

For the three kinds of equations used, the linear equation presents the biggest approximation error with small variation of coefficients. The cubic equations presented the smallest approximation error, but its coefficients varied too much. The quadratic equations presented an intermediate behavior. Note that the approximation errors given by Table 3 are those between the *w*'s curves and the *w*'s values obtained from the optimization problem defined in equations (5) and (6). So, this error is not the approximation error between the displacements measured in tests and those given by the integration of neutral line equation.

One should note that the sections where the largest stiffness loss was numerically obtained, section 17 for the 30 *m* long structure (Fig. 7), were the sections that indeed collapsed in the tests. It is possible that this analysis methodology may be used as a "warning" that a certain section is the most probable to collapse. The formulation presented here is very useful to analyze the behavior of one specific section, especially in failure prevention and maintenance of structures.

Others important considerations here are related to the elasticity modulus of concrete. In this work we considered $E = 41.4$ *GPa*, computed according to NBR-6118-78 (ABNT [2]) Brazilian Code. This value is larger than values measured in tests, around 21 *GPa*, and larger than the value given by the revision of that Code, the new NBR-6118-03 (ABNT [3]), around 31.9 *GPa*. Tests accomplished using the formulations proposed here showed that when we solve the optimization problem and compute a certain function $w_1(x)$ considering a given elasticity modulus of concrete E_1 and solve the problem again using another value E_2, the new value of w is $w_2(x) = E_1w_1/E_2$, in other words, for a fixed value of x the quantity $E_1w_1 = E_2w_2 = E_iw_i$ is a constant in the optimization problems solved here. In our case, the elasticity modulus measured in tests for the structures analyzed here is equal to 21 *GPa*, the coefficients a, b, c e d presented in this work must be multiplied by 41.4/21 = 2.0 to get the correct values of coefficients. In practice, it is not necessary to compute displacements again, because if we use the effective stiffness concept, as we noted earlier, the product Ew stays constant, for a fixed x, and final results are the same.

5 Conclusions

In this work we presented experimental work conducted on 30 *m* above ground long structure. Data from tests were processed using optimization techniques to compute the effective stiffness of sections. We presented one formulation to determine relationships between the effective bending stiffness and the bending moment level in a RC section. This formulation provides one equation for each section of a discretized structure. It can provide information specifically about one given section. It makes possible to predict what section will probably collapse during a possible structural failure and what section is necessary to improve the resistance in case of the need of reinforce an existing structure.

We used three different equations to represent the effective bending stiffness parameter. They are linear, quadratic and cubic equations. In most cases analyzed here the cubic provides the smaller approximation error, but the coefficients of the equations vary considerably from one type of structure to the other, or from one section to the other. Among these equations, the linear one usually provides the largest approximation error but the values of coefficients do not change much from one structure to other or from one section to other.

Other important conclusion is related to concrete elasticity modulus. We noted that when we solve the problem considering different values for the concrete elasticity modulus, E_1, E_2, E_i, and obtain the corresponding functions w_1, w_2, w_i, for a fixed value of x the quantity $E_1w_1(x) = E_2w_2(x) = E_iw_i(x)$ is a constant, in the optimization problems solved here.

The conclusions presented here are based on the analysis of only one 30 *m* long structure. This structure is flanged and, near to the flanges, there are large concentration of steel, what can make this kind of structure stiffer than others non flanged RC structures.

We suggest for future works to analyze several kind of structures, flanged and non flanged, to identify standard equations for each kind of structure. Additional suggestions for future work are:

i) explore some aspects related to the theory of structural failure, searching for a relationship between the sections where the largest losses happen and the sections that indeed collapsed in real similar cases;

ii) in the experimental work, study the influences of the structure self-weight, the friction between the movable supports and steel plates on which they are mounted on and the stiffness of the structure support in the results obtained;

iii) define an optimization problem to minimize the cost of a Telecommunication Tower; use several different kind of *w*'s equations and analyze the sensitivity of the final design related to the coefficients of *w*'s equations.

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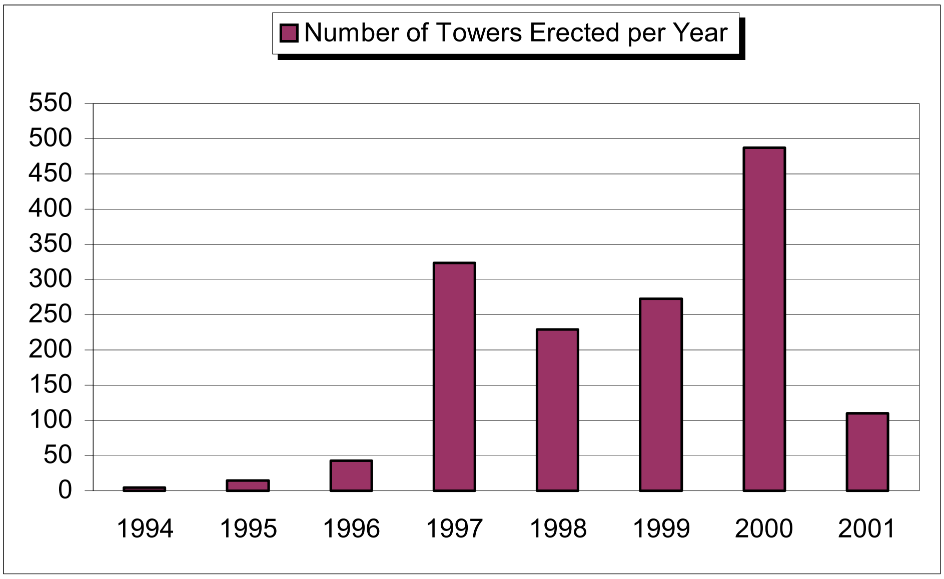

Fig. 1 – Number of towers erected by Scac per year

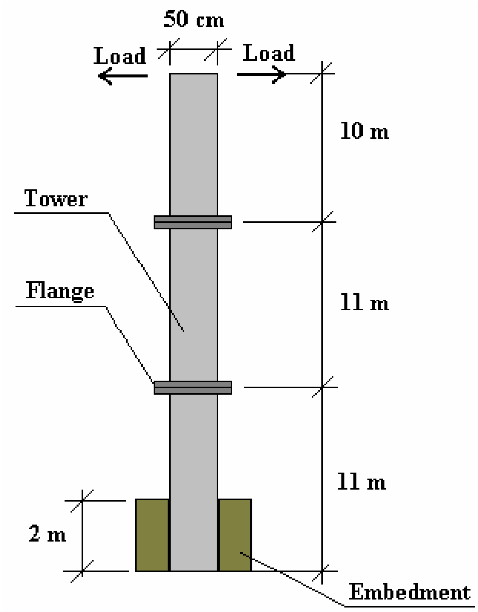

Fig. 2 - Outline of 30 *m* Structure Assembly

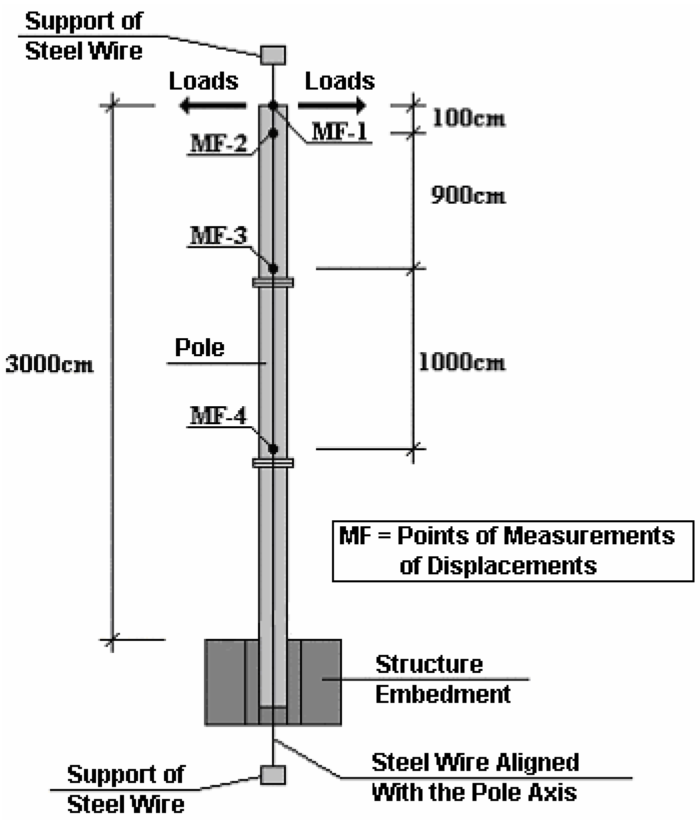

Fig. 3 - Outline of the 30 *m* long structure assembly

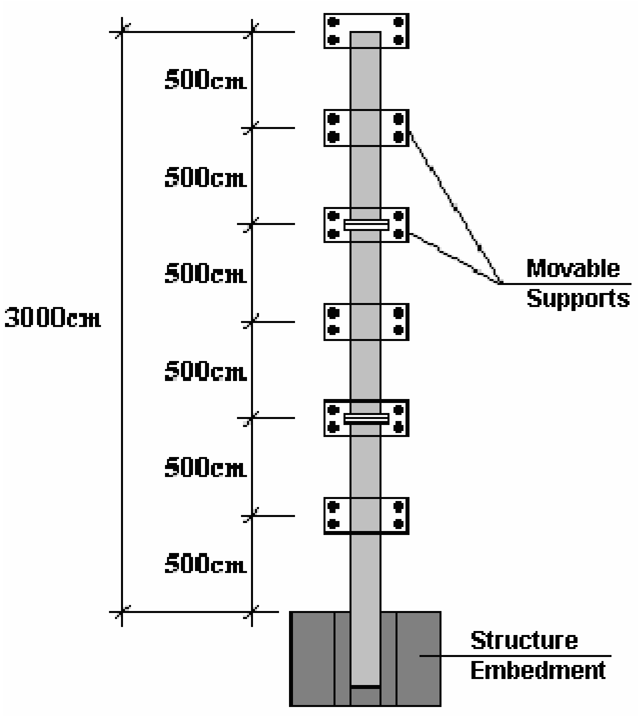

Fig. 4 – Movable supports installed for the 30 *m* long structure

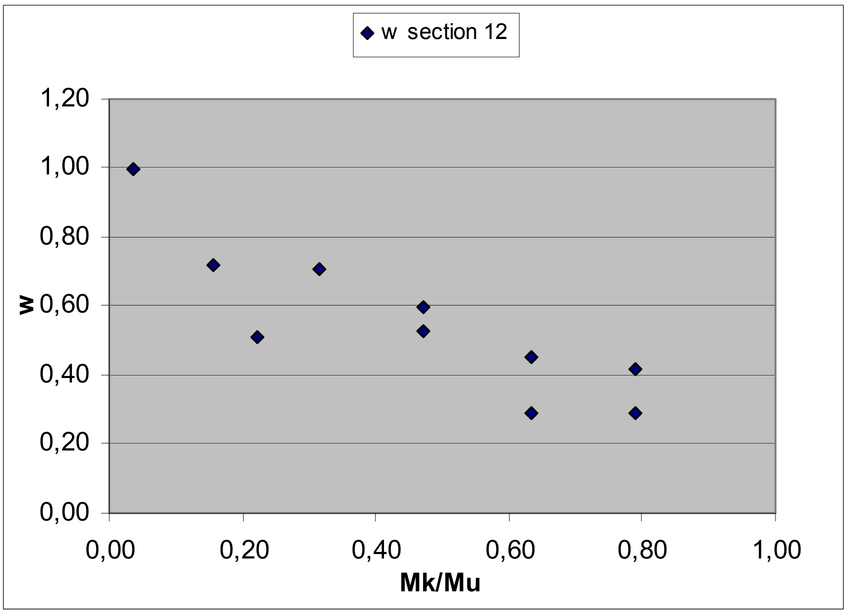

Fig. 5 - Points obtained for section 12 for the 30 *m* long structure

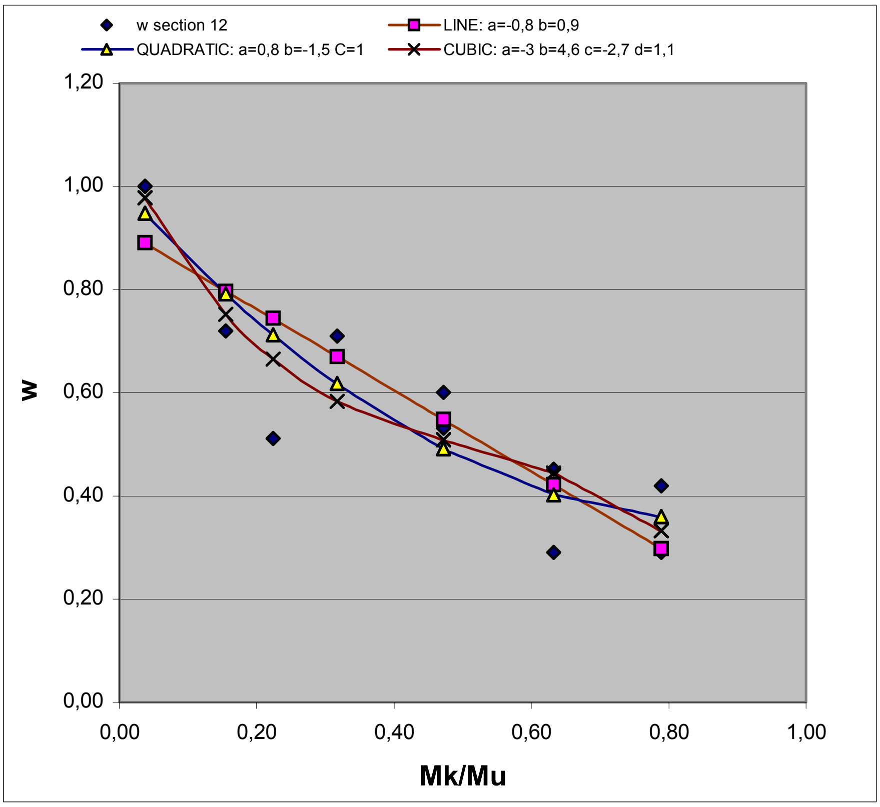

Fig. 6 - Tendencies obtained for section 12 for the 30 *m* long structure

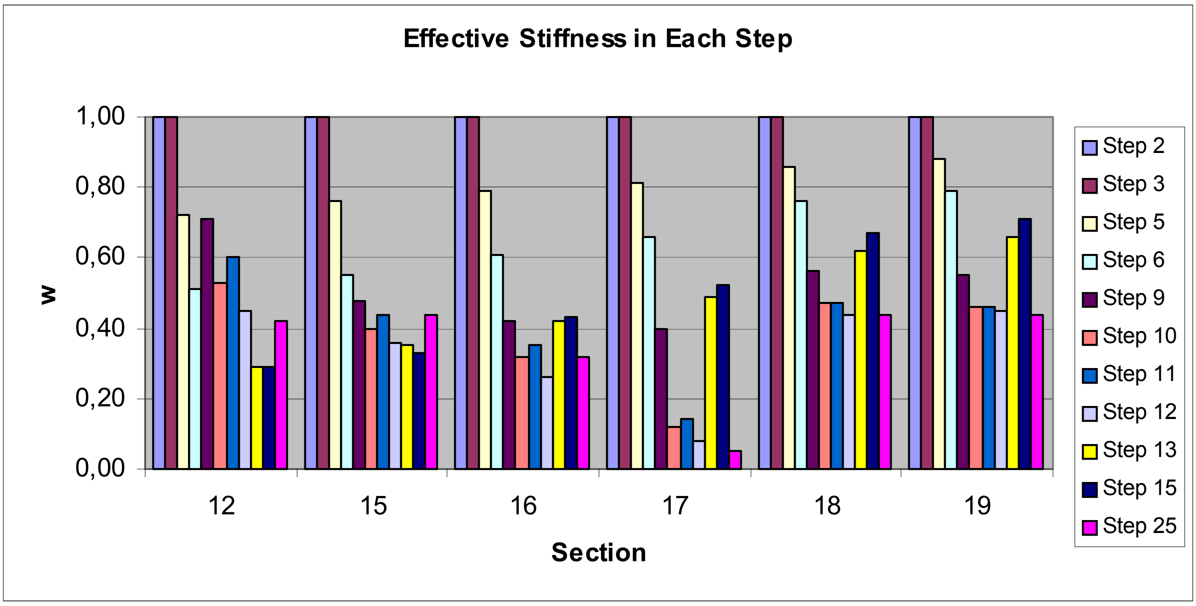

Fig. 7 – Effective stiffness in each step around the section that indeed collapsed (section 17)

Table 1 - Geometrical and steel area characteristics of the 30 *m* above ground long structure

Section	Height (m)	ϕext (cm)	thick. (cm)	A total (cm4)	Ic total (cm4)	As (cm2)	Itotal (cm4)	ws
30	30,00	50,00	9,56	1215	262161	18,1	283414	0,07
29	29,00	50,00	9,56	1215	262161	18,1	283414	0,07
28	28,00	50,00	9,56	1215	262161	18,1	283414	0,07
27	27,00	50,00	9,56	1215	262161	18,1	283414	0,07
26	26,00	50,00	9,56	1215	262161	18,1	283414	0,07
25	25,00	50,00	9,56	1215	262161	18,1	283414	0,07
24	24,00	50,00	9,56	1215	262161	18,1	283414	0,07
23	23,00	50,00	9,56	1215	262161	18,1	283414	0,07
22	22,00	50,00	9,56	1215	262161	18,1	283414	0,07
21	21,00	50,00	9,56	1215	262161	18,1	283414	0,07
20	20,00	50,00	12,66	1485	288584	18,0	309762	0,07
19	19,00	50,00	12,75	1492	289110	42,2	338005	0,14
18	18,00	50,00	12,84	1499	289624	48,3	345546	0,16
17	17,00	50,00	11,16	1362	277980	26,1	308271	0,10
16	16,00	50,00	11,16	1362	277980	28,1	310601	0,11
15	15,00	50,00	11,16	1362	277980	30,2	312931	0,11
14	14,00	50,00	11,16	1362	277980	30,2	312931	0,11
13	13,00	50,00	11,16	1362	277980	32,2	315261	0,12
12	12,00	50,00	11,16	1362	277980	34,2	317591	0,12
11	11,00	50,00	11,16	1362	277980	36,2	319921	0,13
10	10,00	50,00	11,16	1362	277980	40,2	324582	0,14
9	9,00	50,00	12,84	1499	289624	80,4	381128	0,24
8	8,00	50,00	14,97	1647	298832	87,3	398199	0,25
7	7,00	50,00	17,09	1767	303722	94,2	410952	0,26
6	6,00	50,00	12,29	1456	286300	50,3	343490	0,17
5	5,00	50,00	12,29	1456	286300	53,4	347064	0,18
4	4,00	50,00	12,29	1456	286300	53,4	347064	0,18
3	3,00	50,00	12,29	1456	286300	56,5	350638	0,18
2	2,00	50,00	12,29	1456	286300	59,7	354213	0,19
1	1,00	50,00	12,29	1456	286300	62,8	357787	0,20
0	0,00	50,00	12,29	1456	286300	62,8	357787	0,20

Table 2 - Loads and measured displacements of the 30 *m* above ground long structure

Step	Load (N)		Displacements (cm)						
	Value	Orientation	30m	29m	25m	20m	15m	10m	5m
1	600	right	0,1	0,0	0,0	0,0	0,0	0,0	0,0
2	600	left	0,2	0,2	0,0	0,0	0,0	0,0	0,0
3	2500	left	20,1	18,8	14,8	10,0	5,8	2,9	0,6
4	3600	left	35,0	32,8	25,9	17,3	10,0	5,2	1,5
5	5100	left	46,7	44,1	34,4	23,0	13,6	6,8	1,9
6	7600	right	93,0	87,9	68,0	45,4	26,3	12,6	4,1
7	7600	left	100,5	95,2	74,9	51,1	30,7	15,7	4,7
8	10200	right	163,1	154,7	120,4	80,9	47,8	23,1	7,2
9	12700	left	243,6	231,9	184,7	129,6	80,9	48,7	14,9
10	12700	right	214,7	203,2	157,1	104,2	60,1	28,1	8,8

Table 3 - Values of *a*, *b*, *c* and *d* for sections solicited up to $0.8M_u$, for the 30 *m* long structure

Coefficients		**Sections**																**Mean**
		0	**1**	**2**	**3**	**4**	**5**	**6**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**21**	
LINE	**a**	-0,8	-0,9	-1,1	-1,1	-0,6	-0,6	-0,6	-0,6	-0,7	-0,8	-0,8	-0,8	-0,8	-0,9	-1,1	-0,6	**-0,8**
	b	1,0	0,9	1,0	1,0	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	0,9	**0,9**
	Error	0,05	1,42	0,33	0,17	0,50	0,39	0,43	0,79	0,74	0,61	0,51	0,60	0,73	1,02	2,54	1,17	**0,75**
QUADRATIC	**a**	0,1	2,0	0,8	0,8	1,2	0,5	0,4	0,6	0,7	0,8	1,0	1,4	2,1	2,6	3,1	2,4	**1,3**
	b	-0,9	-2,5	-1,7	-1,6	-1,5	-1,0	-0,9	-1,1	-1,3	-1,5	-1,7	-2,0	-2,5	-2,9	-3,5	-2,4	**-1,8**
	c	1,0	1,1	1,1	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,1	1,1	1,2	1,1	**1,0**
	Error	0,05	0,82	0,23	0,14	0,28	0,36	0,40	0,74	0,64	0,49	0,33	0,25	0,10	0,14	1,26	0,51	**0,42**
CUBIC	**a**	-0,6	-0,4	0,5	-5,3	-1,5	-6,2	-5,6	-3,4	-2,8	-3,0	-3,1	-3,1	-2,5	-1,0	1,7	1,3	**-2,2**
	b	0,9	2,5	0,2	6,0	3,0	7,8	7,1	4,8	4,2	4,6	4,9	5,3	5,1	3,8	1,2	1,0	**3,9**
	c	-1,1	-2,7	-1,5	-2,9	-2,1	-3,2	-3,0	-2,4	-2,4	-2,7	-3,0	-3,2	-3,4	-3,2	-2,9	-2,0	**-2,6**
	d	1,0	1,1	1,0	1,1	1,1	1,1	1,1	1,0	1,1	1,1	1,1	1,1	1,1	1,1	1,1	1,1	**1,1**
	Error	0,05	0,82	0,23	0,11	0,27	0,20	0,23	0,67	0,59	0,42	0,26	0,18	0,07	0,13	1,25	0,50	**0,37**